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EMERGENCE OF POWER-LAW IN SPATIAL EPIDEMICS USING CELLULAR AUTOMATION

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We analyze a spatial susceptible-infected epidemic model using cellular automata and investigate the relations between the power-law distribution of patch sizes and the regime of invasion. The obtained results show that, when the invasion is in the form of coexistence of stable target and spiral wave, power-law will emerge, which may provide a new insight into the control of disease.

Keywords: Cellular automata; epidemiology; power-law; wave.

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1. Introduction

One main approach, used to model the disease spreading, is based on deterministic, time-continuous partial differential equations, where the susceptible and infected density are considered as real, continuous variables.[1–4] Such models can support either homogeneous or patchy solutions, where the resulting invasion form of disease are either regular[1,2,4–7] or irregular.[3] These models, however, fail to capture certain important features, such as the distribution of patch sizes; the typical size of a patch is dictated by either the model parameters (in the Turing case) or the initial conditions (for subcritical bifurcation, where two metastable solutions may exist simultaneously in the system, and the domains structure depends on system's history).[8] Moreover, they sometimes come at the expense of various simplifying assumptions that limit their applicability, in particular in evolutionary contexts.

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The advent of modern computing facilities has encouraged epidemiologists to turn to individual-based spatial simulations, such as cellular automata.[9,10] In cellular automata, space is represented by a network of sites, and the state of the network is updated through probabilistic rules representing demographic events.[11]

Some scholars observed the epidemic data, which suggested that patch sizes in the semiarid zone obey, at least in some parameter regions studied, power-law distributions.[12–16] In the present paper, we do not intend to solve the power-law mystery here; instead, let us show that our epidemic model may actually support the same behavior, especially reveal what is the invasion form of disease when power-law emerges.

2. Method

A spatial susceptible-infected model, with birth and death events, will be illustrated. Generally speaking, the population, in which a pathogenic agent is active, comprises two subgroups: the healthy individuals who are susceptible (S) and the already infected individuals (I).[17,18] Also, there is another state which is the cellular automation-empty state (E). Here, eight neighbors are used.

Within a subpopulation, the dynamics for the local populations obey a basic reaction scheme conserving the number of population, which has been studied both in physics and mathematical epidemiology, namely the infection dynamics process identified by the following set of reactions

$$S + I \xrightarrow{\beta} I + I\,, \tag{1a}$$

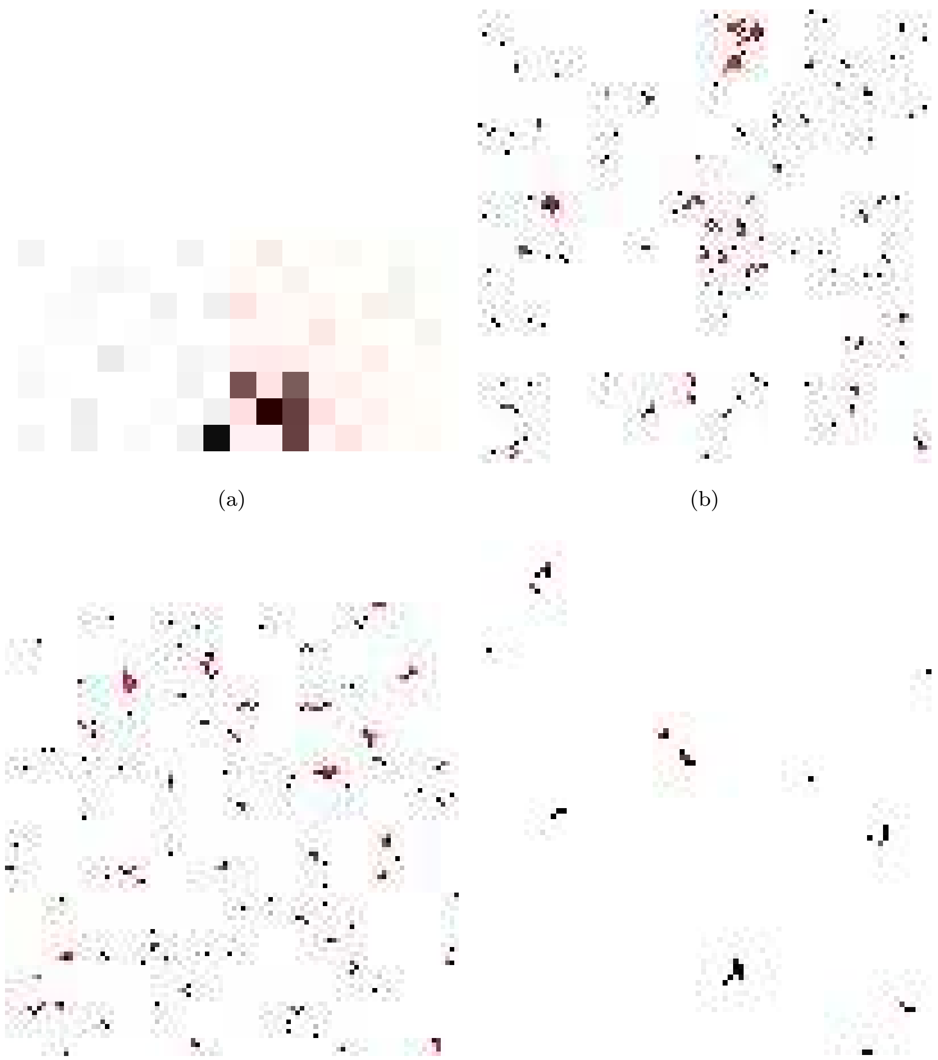
$$I \xrightarrow{d} E\,, \tag{1b}$$

$$E + S \xrightarrow{b} S + S\,. \tag{1c}$$

In the cellular automation, S, I, and E mean the state in one discrete lattice. The first reaction [Eq. (1a)] reflects the fact that an infected host (I) can infect susceptible (S) neighbors with infection rate $1 - (1 - \beta)^{N_I}$, where N_I is the sum of infected in the neighborhood; the second reaction [Eq. (1b)] indicates the infected host (I) will die with death rate d; and the third reaction [Eq. (1c)] indicates that the empty state becomes occupied by the offspring reproduced from the susceptible neighbors with birth rate $1 - (1 - b)^{N_S}$, where N_S is the sum of susceptible in the neighborhood.

3. Main Results

In this section, we will give the numerical simulations of the spatial SI model in two-dimensional space. The grid sizes in the evolutional simulations are $m \times n$ (here $m = n = 200$). Biological invasion usually starts with a local introduction of exotic species; thus, relevant initial conditions for system (1) should be described by functions of compact support when the density of one or both populations at the initial moment of time is nonzero only inside a certain domain. The shape of the domain and the profiles of the population densities can be different in different cases.[19] In order to study the biological invasion in the model (1), the initial distribution of populations was taken as follows: $S(x, y, 0) = S_0$, if $x_1 < x < x_2$ and $y_1 < y < y_2$ (here, $x_1 = 30$, $x_2 = 90$, $y_1 = 50$, and $y_2 = 110$), otherwise



(a) (b) (c) (d)

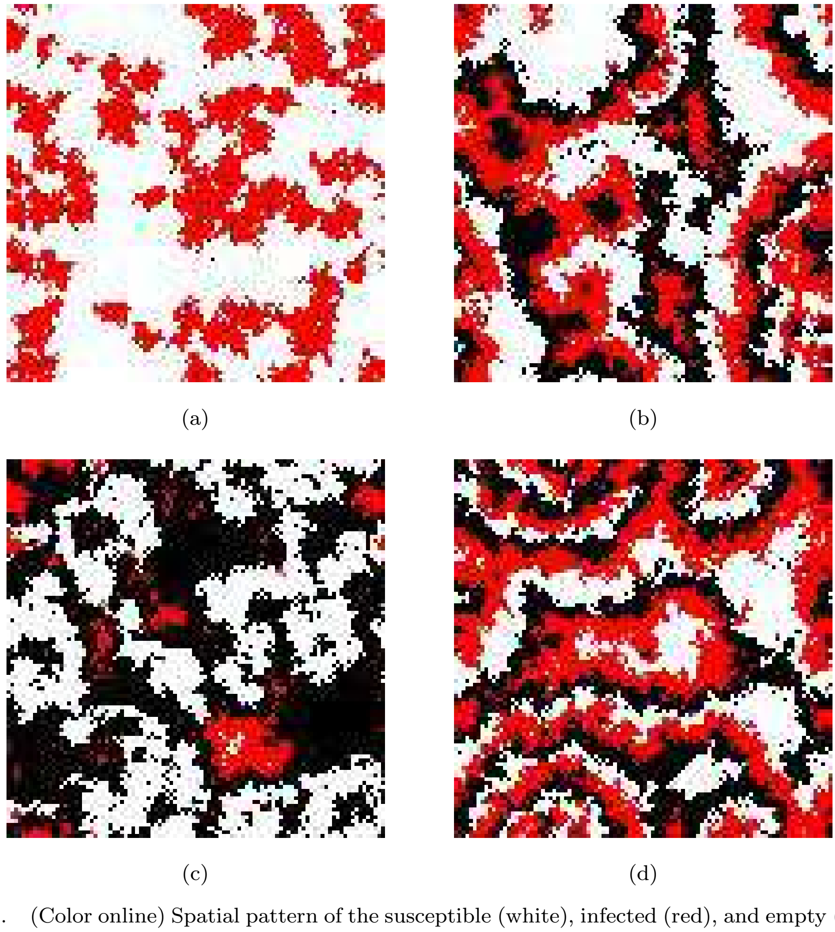
Fig. 1. (Color online) Spatial pattern of the susceptible (white), infected (red), and empty (black) for different time. All panels are depicted on a 200×200 spatial grid. Parameter values used as: $\beta = 0.3$, $b = 0.5$, and $d = 0.5$. (a) $t = 50$; (b) $t = 150$; (c) $t = 500$; (d) $t = 1000$.

$S(x, y, 0) = 0$; $I(x, y, 0) = I_0$, if $x_3 < x < x_4$ and $y_3 < y < y_4$ (here, $x_3 = 40$, $x_4 = 100$, $y_3 = 40$, and $y_4 = 100$), otherwise $I(x, y, 0) = 0$.

In Fig. 1, spatial patterns of infected population at $t = 50$, 150, 500, and 1000 with $S_0 = I_0 = 0.1$ and small values of β are presented. One can see from this figure that, the disease number exhibits oscillation behavior and reaches its maximum value at $t \approx 150$.

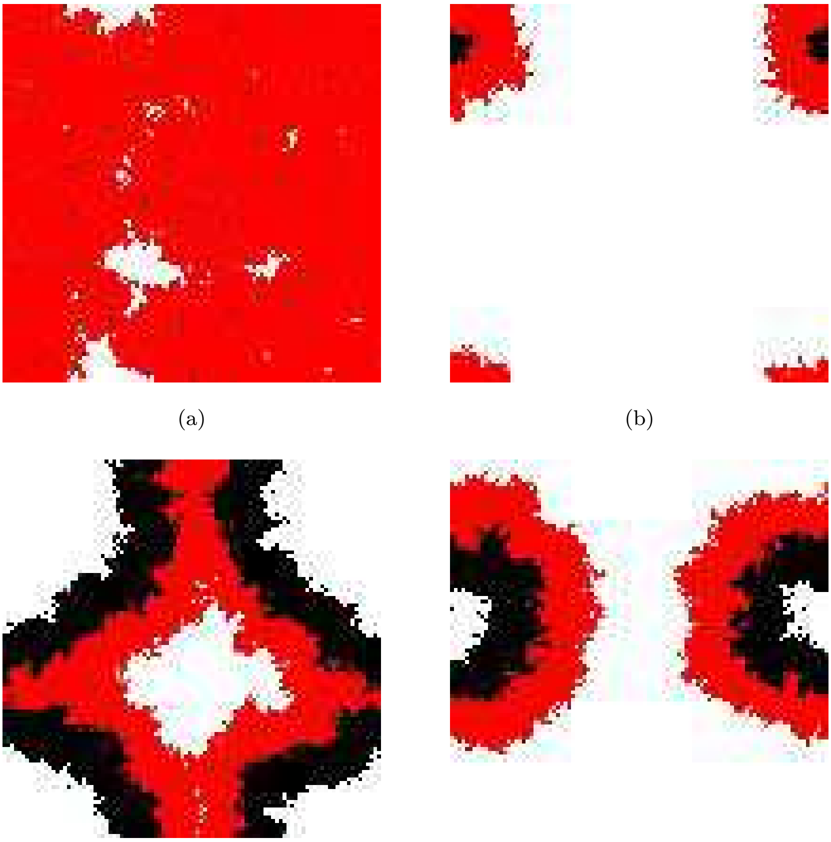
In Fig. 2, it shows the evolution of the spatial pattern of infected population at $t = 50$, 150, 500, and 1000, with the same initial conditions in Fig. 1 but $\beta = 2$. It can be seen from this figure that, stable target and spiral wave can coexist in the two-dimensional space, which means the infected are in the form of high density and can ensure the persistence of the disease.

In Fig. 3, it shows the evolution of the spatial pattern of infected population at $t = 50$, 150, 500, and 1000, with the same initial conditions in Fig. 1 but $\beta = 5$.



(a) (b) (c) (d)

Fig. 2. (Color online) Spatial pattern of the susceptible (white), infected (red), and empty (black) for different time. All panels are depicted on a 200×200 spatial grid. Parameter values used as: $\beta = 2$, $b = 0.5$, and $d = 0.5$. (a) $t = 50$; (b) $t = 150$; (c) $t = 500$; (d) $t = 1000$.



(a) (b) (c) (d)

Fig. 3. (Color online) Spatial pattern of the susceptible (white), infected (red), and empty (black) for different time. All panels are depicted on a 200 × 200 spatial grid. Parameter values used as: $\beta = 5$, $b = 0.5$, and $d = 0.5$. (a) $t = 50$; (b) $t = 150$; (c) $t = 500$; (d) $t = 1000$.

High density of the infected can be seen from this figure. As time increases, two target waves form in the spatial domain.

We tested the cumulative distribution function for the cluster sizes, $P(A > \alpha)$ defined as the probability for an infected patch to have an area greater than α. And we found that, only when the invasion form of disease is in the form of coexistence of stable target and spiral wave (see Fig. 2), a power-law $P(A > \alpha) = \kappa\alpha^{-\gamma}$ seems to fit the results, which we show in Fig. 4.

4. Discussion and Conclusion

In the field of epidemiology, the power-law has been found by Rhodes and Anderson.[12] From then on, some papers report on power-law behaviors.[13–16] However, the corresponding invasion form is not well revealed. In order to well reveal the mechanism, an epidemic model using cellular automata with both birth and death rate

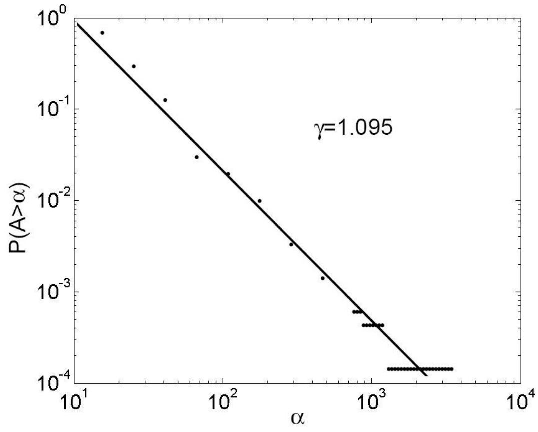


Fig. 4. The cluster size distribution shown for the infected in a log–log plot. Parameter values used as: $\beta = 2$, $b = 0.5$, and $d = 0.5$. The simulations used a 200×200 lattice, and the same initial conditions are the same as in Fig. 2.

is presented. We find that, the cluster size of the infected shows a power-law distribution, when the invasion form is in the coexistence of spiral and target wave. The obtained results may have implications for how we try to prevent, and eventually eradicate, the disease.

The power-law distributions occur in many situations of scientific interest. The present paper provides some new insights into the spatial epidemiology, but it also leaves many questions open for the future investigations. I would like to emphasize two relevant topics which may be the main subjects of the further research. Firstly, how to use the power-law to characterize multistrain evolutionary dynamics and spatial interaction? Secondly, is there an example of the epidemics outbreaks as a power-law form in space?

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