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Strong matching preclusion for k-ary n-cubes[☆]

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ABSTRACT

The k-ary n-cube is one of the most popular interconnection networks for parallel and distributed systems. Strong matching preclusion that additionally permits more destructive vertex faults in a graph is a more extensive form of the original matching preclusion that assumes only edge faults. In this paper, we establish the strong matching preclusion number and all minimum strong matching preclusion sets for k-ary n-cubes with $n \geq 2$ and $k \geq 3$.

1. Introduction

A matching of a graph is a set of pairwise nonadjacent edges. For a graph with n vertices, a matching M is called perfect if its size $|M| = \frac{n}{2}$ for even n, or almost perfect if $|M| = \frac{n-1}{2}$ for odd n. A graph is matchable if it has either a perfect matching or an almost perfect matching. Otherwise, it is called unmatchable. Throughout the paper, we only consider simple graphs, that is, graphs with no parallel edges or loops. For graph-theoretical terminology and notation not defined here, we follow [3]. Let $G = (V(G), E(G))$ be a graph. A set F of edges in G is called a matching preclusion set (MP set for short) if $G - F$ has neither a perfect matching nor an almost perfect matching. The matching preclusion number of G (MP number for short), denoted by $mp(G)$, is defined to be the minimum size of all possible such sets of G. The minimum MP set of G is any MP set whose size is $mp(G)$. A matching preclusion set of a graph is trivial if all its edges are incident to a single vertex.

Since the problem of matching preclusion was first presented by Brigham et al. [6], several classes of graphs have been studied to understand their matching preclusion properties [8–11,15,17,19]. An obvious application of the matching preclusion problem was addressed in [6]: when each node of interconnection networks is required to have a special partner at any time, those that have larger matching preclusion numbers will be more robust in the event of link failures.

Another form of matching obstruction, which is in fact more offensive, is through vertex failures. As an extensive form of matching preclusion, the problem of strong matching preclusion was proposed by Park and Ihm in [16]. A set F of vertices and/or edges in a graph G is called a strong matching preclusion set (SMP set for short) if $G-F$ has neither a perfect matching nor an almost perfect matching. The strong matching preclusion number (SMP number for short) of G, denoted by $smp(G)$, is defined to be the minimum size of all possible such sets of G. The minimum SMP set of G is any SMP set whose size is $smp(G)$. Note that the strong matching preclusion is more general than the problems discussed in [1,13], which considered only vertex deletions.

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In particular, when G itself does not contain perfect matchings or almost perfect matchings, both $smp(G)$ and $mp(G)$ are regarded as zero. These numbers are undefined for a trivial graph with only one vertex. Notice that an MP set of a graph is a special SMP set of the graph.

Proposition 1.1 ([16]). *For every nontrivial graph G, $smp(G) \leq mp(G)$.*

However, the strong matching preclusion numbers did not decrease for such graphs as restricted hypercube-like graphs and recursive circulants [16]. Following this work, the strong matching preclusion problem was studied for some classes of graphs such as alternating group graphs, split-stars, and augmented cubes [4,12].

When a set F of vertices and/or edges is removed from a graph, the set is called a fault set. Let F_v and F_e be the fault vertex set and the fault edge set, respectively. We have $F = F_v \cup F_e$. For any vertex $v \in V(G)$, let $N_G(v)$ be all neighboring vertices adjacent to v, and let $I_G(v)$ be all edges incident to v. Clearly, a fault set that separates exactly one isolated vertex from the remaining even graph forms a simple SMP set of the original graph.

Proposition 1.2 ([16]). *Let G be a graph. Given a fault vertex set $X(v) \subseteq N_G(v)$ and a fault edge set $Y(v) \subseteq I_G(v)$, $X(v) \cup Y(v)$ is an SMP set of G if* (i) *$w \in X(v)$ if and only if $(v, w) \notin Y(v)$ for every $w \in N_G(v)$, and* (ii) *the number of vertices in $G - (X(v) \cup Y(v))$ is even.*

The above proposition suggests an easy way of building SMP sets. Any SMP set constructed as specified in Proposition 1.2 is called trivial. If $smp(G) = \delta(G)$, then G is called maximally strong matched. If every minimum SMP set of G is trivial, then G is called super strong matched. It is easy to see that, for an arbitrary vertex of degree at least 1, there always exists a trivial SMP set which isolates the vertex. This observation leads to the following fact.

Proposition 1.3 ([16]). *For any graph G with no isolated vertices, $smp(G) \leq \delta(G)$, where $\delta(G)$ is the minimum degree of G.*

2. Definitions and terminology

The k-ary n-cube, denoted by Q_n^k, as one of the most attractive interconnection networks for parallel and distributed systems, has drawn considerable research attention for its desirable properties [7,19,20], such as ease of implementation, low latency, and high-bandwidth inter-processor communication [5]. Several parallel systems, such as iWarp [18], Cray T3D [14], and Cray T3E [2], have been built based on the k-ary n-cube. The k-ary n-cube Q_n^k ($k \geq 2$ and $n \geq 1$) is a graph consisting of k^n vertices, each of which has the form $u = \delta_1\delta_2 \ldots \delta_n$, where $0 \leq \delta_i \leq k-1$ for $1 \leq i \leq n$. Two vertices $u = \delta_1\delta_2 \ldots \delta_n$ and $v = \lambda_1\lambda_2 \ldots \lambda_n$ are adjacent if and only if there exists an integer j, $1 \leq j \leq n$, such that $\delta_j = \lambda_j \pm 1 (\text{mod } k)$ and $\delta_i = \lambda_i$, for every $i \in \{1, 2, \ldots, n\} \setminus \{j\}$. Such an edge (u, v) is called a j-dimensional edge. For clarity of presentation, we omit writing "(mod k)ïn similar expressions for the remainder of the paper. Note that each vertex has degree $2n$ when $k \geq 3$, and degree n when $k = 2$. Obviously, Q_1^k is a cycle of length k, and Q_n^2 is an n-dimensional hypercube. We say that Q_n^k is divided into $Q_n^k[0], Q_n^k[1], \ldots, Q_n^k[k-1]$ (abbreviated as $Q[0], Q[1], \ldots, Q[k-1]$, if there are no ambiguities) along dimension d for some $1 \leq d \leq n$, where $Q[p]$, for every $0 \leq p \leq k-1$, is a subgraph of Q_n^k induced by $\{u = \delta_1\delta_2 \ldots \delta_d \ldots \delta_n \in V(Q_n^k) : \delta_d = p\}$. It is clear that each $Q[p]$ is isomorphic to Q_{n-1}^k for $0 \leq p \leq k-1$. Let $u_i = \delta_1\delta_2 \ldots \delta_{d-1} i \delta_{d+1} \ldots \delta_n$ be an arbitrary vertex of $Q[i]$. For $j \in \{0, 1, \ldots, k-1\} \setminus \{i\}$, the vertex $u_j = \delta_1\delta_2 \ldots \delta_{d-1} j \delta_{d+1} \ldots \delta_n$ is called the corresponding vertex of u_i in $Q[j]$.

A graph is bipartite if its vertex set can be partitioned into two subsets X and Y such that every edge has one end in X and one end in Y. Denote by $|G|$ the number of vertices in a graph G. A path is a simple graph whose vertices can be arranged in a linear sequence in such a way that two vertices are adjacent if they are consecutive in the sequence, and are nonadjacent otherwise. The length of a path is the number of its edges. The path is odd or even according to the parity of its length. Let G_1 and G_2 be two graphs. $G_1 \cup G_2$ is the graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. A path or a cycle which contains every vertex of a graph is called a Hamiltonian path or Hamiltonian cycle of the graph. A graph is Hamiltonian if it has a Hamiltonian cycle. A graph G is Hamiltonian connected if, for two arbitrary vertices x and y in G, there is a Hamiltonian path connecting x and y. Let F be a faulty set in a graph G which contains vertices and/or edges. Let k be a positive integer. G is k-Hamiltonian if $G - F$ is Hamiltonian for every F with $|F| \leq k$. G is k-Hamiltonian connected if $G - F$ is Hamiltonian connected for every F with $|F| \leq k$.

In this paper, we investigate the problem of strong matching preclusion for k-ary n-cubes. We shall establish the strong matching preclusion number and all possible minimum strong matching preclusion sets for k-ary n-cubes with $n \geq 2$ and $k \geq 3$.

3. Main results

We first study strong matching preclusion for Q_1^k. Recall that Q_1^k is a cycle of length k when $k \geq 3$. We have the following result.

Theorem 3.1. *Let $k \geq 3$ be an integer. Then $smp(Q_1^k) = 2$.*

Proof. By Proposition 1.3, $smp(Q_1^k) \leq \delta(Q_1^k) = 2$. Next, consider a fault set F with $|F| = 1$. If F consists of one edge, $Q_1^k - F$ is a path of length $k-1$. If F consists of one vertex, $Q_1^k - F$ is a path of length $k-2$. Note that an odd path has a perfect matching, while an even path has an almost perfect matching. We have that $Q_1^k - F$ is matchable, which means that $smp(Q_1^k) > 1$. Therefore, $smp(Q_1^k) = 2$. □

By Theorem 3.1, Q_1^k is maximally strong matched, where $k \geq 3$. However, Q_1^k is not super strong matched. For example, let $F = \{(0, 5), (2, 3)\}$ be the fault set in a Q_1^6. It is easy to see that there is no perfect matching in $Q_1^6 - F$ and that F is not a trivial strong matching preclusion set.

Lemma 3.1 ([16]). *For a connected m-regular bipartite graph G with $m \geq 3$, $smp(G) = 2$. Furthermore, each of its minimum SMP sets is a set of two vertices from the same partite set.*

Theorem 3.2. *Let $k \geq 4$ be an even integer, and let $n \geq 2$ be an integer. Then Q_n^k is bipartite, and $smp(Q_n^k) = 2$. Furthermore, each of its minimum SMP sets is a set of two vertices from the same partite set.*

Proof. Let $V_1 = \{\delta_1\delta_2 \dots \delta_n : \delta_1\delta_2 \dots \delta_n \in V(Q_n^k)$ and $\sum_{i=1}^n \delta_i = 0 (\mathrm{mod}\ 2)\}$ and $V_2 = V(Q_n^k)\backslash V_1$. Without loss of generality, let $\delta_1\delta_2 \dots \delta_n \in V_1$ and $\lambda_1\lambda_2 \dots \lambda_n \in N_{Q_n^k}(\delta_1\delta_2 \dots \delta_n)$. By the definition of Q_n^k, there exists some $j \in \{1, 2, \dots, n\}$ such that $\delta_j = \lambda_j \pm 1 (\mathrm{mod}\ k)$ and $\delta_i = \lambda_i$ for $i \in \{1, 2, \dots, n\}\backslash\{j\}$. Since k is even, δ_j and λ_j have different parities, which implies that $\sum_{i=1}^n \delta_i$ and $\sum_{i=1}^n \lambda_i$ have different parities. So $\lambda_1\lambda_2 \dots \lambda_n \in V_2$. It follows that two arbitrary vertices in V_1 are nonadjacent. Similarly, two arbitrary vertices in V_2 are nonadjacent. Thus, Q_n^k is bipartite. Note that Q_n^k is a connected $2n$-regular graph with $2n > 3$. By Lemma 3.1, $smp(Q_n^k) = 2$, and each of its minimum SMP sets is a set of two vertices from the same partite set. □

Lemma 3.2 ([20]). *Let $n \geq 2$ be an integer, and let $k \geq 3$ be an odd integer. Then Q_n^k is $(2n-2)$-Hamiltonian and is $(2n-3)$-Hamiltonian connected.*

Theorem 3.3. *Let $n \geq 2$ be an integer, and let $k \geq 3$ be an odd integer. Then $smp(Q_n^k) = 2n$.*

Proof. Let $F \subseteq V(Q_n^k) \cup E(Q_n^k)$ with $|F| \leq 2n - 1$. By Lemma 3.2, Q_n^k is $(2n-2)$-Hamiltonian. This implies that $Q_n^k - F$ has a Hamiltonian path. Note that an odd path has a perfect matching, while an even path has an almost perfect matching. Therefore, $Q_n^k - F$ is matchable, which means that $smp(Q_n^k) > 2n - 1$. Combining this with Proposition 1.3, $2n - 1 < smp(Q_n^k) \leq \delta(Q_n^k) = 2n$; that is, $smp(Q_n^k) = 2n$. □

In the following, we shall classify all minimum strong matching preclusion sets for k-ary n-cubes with $n \geq 2$ and odd $k \geq 3$. We claim that all minimum strong matching preclusion sets are trivial. Given the recursive structure of k-ary n-cubes, the natural method is to use induction. The first step is to check the base case by case-by-case analysis. Then, combining the basis of the induction with Hamiltonicity of faulty k-ary n-cubes, we will prove that Q_n^k ($n \geq 2$ and odd $k \geq 3$) is super strong matched by induction on n.

Theorem 3.4. *Let $k \geq 3$ be an odd integer. Then Q_2^k is super strong matched.*

Proof. Q_2^k can be divided into $Q[0], Q[1], \dots, Q[k-1]$ along dimension 1. For $i \in \{0, 1, \dots, k-1\}$, since $Q[i]$ is a cycle of length k, for notational simplicity, denote $Q[i]$ by C_i. Denote the set of 1-dimensional edges between C_i and C_{i+1} by $M_{i,i+1}$ for $0 \leq i \leq k-1$. For any $u_0 \in V(C_0)$, the vertices $u_0, u_1, \dots, u_{k-1}$ and the 1-dimensional edges between them form a cycle of length k, which is denoted by $C(u_i)$ for some $i \in \{0, 1, \dots, k-1\}$. For any matching M_i in C_i, the matching M_j, which satisfies that $(x_j, y_j) \in M_j$ if and only if $(x_i, y_i) \in M_i$, is called the corresponding matching to M_i.

By Theorem 3.3, $smp(Q_2^k) = 4$. Let $F = F_v \cup F_e$ be a fault set in Q_2^k such that $|F| = 4$, where F_v and F_e are the fault vertex set and the fault edge set, respectively. To prove our main result, it is enough to show that either $Q_2^k - F$ is matchable or F is a trivial strong matching preclusion set, where no fault edge in F is incident to any fault vertex in F. We consider five cases, depending on the value of $|F_v|$. Without loss of generality, assume that $|F_v \cap V(C_0)| \geq |F_v \cap V(C_i)|$ for $i = 1, 2, \dots, k-1$.

Case 1. $|F_v| = 4$, which means that $|F_e| = 0$.

Let $F' \subseteq F$ with $|F'| = 2$. By Lemma 3.2, $Q_2^k - F'$ has a Hamiltonian cycle C. Note that $|C| = k^2 - 2$ is odd. $C - (F\backslash F')$ is an even path or is divided into an even path and an odd path. So $C - (F\backslash F')$ can be partitioned into the set M of paths of length 1 plus one single vertex. Now, M is an almost perfect matching in $Q_2^k - F$, which means that $Q_2^k - F$ is matchable.

Case 2. $|F_v| = 3$ and $|F_e| = 1$. Now, there is exactly one fault edge e in Q_2^k.

Case 2.1. $|F_v \cap V(C_0)| = 3$.

Assume that $C_0 - F_v$ can be partitioned into the set M_0 of paths of length 1. Let $M = M_0 \cup M_{1,2} \cup \dots \cup M_{k-2,k-1}$. If $e \notin M$, then M is a perfect matching in $Q_2^k - F$. If $e \in M_0$, say $e = (u_0, v_0)$, then $M \cup \{(u_0, u_1), (v_0, v_1), (u_2, v_2)\}\backslash \{(u_0, v_0), (u_1, u_2), (v_1, v_2)\}$ is a perfect matching in $Q_2^k - F$. If $e \in M\backslash M_0$, say $e = (u_j, u_{j+1})$, then $M \cup \{(u_j, v_j), (u_{j+1}, v_{j+1})\}\backslash \{(u_j, u_{j+1}), (v_j, v_{j+1})\}$ is a perfect matching in $Q_2^k - F$, where $v_j \in N_{C_j}(u_j)$.

Assume that $C_0 - F_v$ can be partitioned into the set M_0 of paths of length 1 plus two single vertices u_0 and v_0, both of which are adjacent to one of the fault vertices, say z_0. Note that $|F_e| = 1$. Without loss of generality, assume that (u_0, u_1)

and (v_0, v_{k-1}) are not fault edges. Let M_1 and M_{k-1} be the perfect matchings of $C_1 - u_1$ and $C_{k-1} - v_{k-1}$, respectively. Let $M = M_0 \cup M_1 \cup M_{k-1} \cup M_{2,3} \cup \cdots \cup M_{k-3,k-2} \cup \{(u_0, u_1), (v_0, v_{k-1})\}$. If $e \notin M$, then M is a perfect matching in $Q_2^k - F$. If $e \in M \backslash (M_0 \cup M_1 \cup M_{k-1})$, say $e = (s_j, s_{j+1})$, then $M \cup \{(s_j, t_j), (s_{j+1}, t_{j+1})\} \backslash \{(s_j, s_{j+1}), (t_j, t_{j+1})\}$ is a perfect matching in $Q_2^k - F$, where $t_j \in N_{C_j}(s_j)$. Next, suppose that $e \in M_0 \cup M_1 \cup M_{k-1}$. Let $F_v \backslash \{z_0\} = \{x_0, y_0\}$. Let M_z be the perfect matching in $C(z) - z$ for each $z \in \{x_0, y_0\}$. Let M_i be the corresponding matching to M_0 for $i = 1, 2, \ldots, k-1$. Let M_{t_0} be the matching saturating $C(t_0) - \{t_0, t_{k-1}, t_1\}$ for each $t_0 \in \{u_0, v_0, z_0\}$. Note that $C = (u_0, u_1, z_1, v_1, v_0, v_{k-1}, z_{k-1}, u_{k-1}, u_0)$ is a cycle which contains at most one fault edge. So there exists a perfect matching M^* containing no fault edges of C. Let $M' = (\cup_{i=0}^{k-1} M_i) \cup (\cup_{t_0 \in \{u_0, v_0, x_0, y_0, z_0\}} M_{t_0}) \cup M^*$. Clearly, there is no fault edge in $(\cup_{t_0 \in \{u_0, v_0, x_0, y_0, z_0\}} M_{t_0}) \cup M^*$. If $e \notin \cup_{i=0}^{k-1} M_i$, then M' is a perfect matching in $Q_2^k - F$. If $e \in M_j \subseteq \cup_{i=0}^{k-1} M_i$, say $e = (a_j, b_j)$, then $M' \cup \{(a_j, a_{j+1}), (b_j, b_{j+1})\} \backslash \{(a_j, b_j), (a_{j+1}, b_{j+1})\}$ is a perfect matching in $Q_2^k - F$.

Case 2.2. $|F_v \cap V(C_0)| = 2$.

Let $F_v \cap V(C_0) = \{u_0, v_0\}$. Let $x_i \in V(C_i)$ be the fault vertex which is not in C_0. $C_0 - \{u_0, v_0\}$ can be divided into an odd path P_0 and an even path P_1. We consider two subcases.

Case 2.2.1. $i \in \{1, k-1\}$. Without loss of generality, assume that $i = 1$.

Assume that $|P_1| = 1$, say $P_1 = y_0$. First, consider that $e = (y_0, y_{k-1})$. If $y_0 = x_0$, then F is a trivial strong matching preclusion set. If $y_0 \neq x_0$, then $C_1 - \{x_1, y_1\}$ can be divided into an odd path P_0' and an even path P_1'. Let z_1 be a terminal vertex of P_1'. Let M_1 and M_2 be the perfect matchings of $C_1 - \{x_1, y_1, z_1\}$ and $C_2 - z_2$, respectively. Let M_0 be a perfect matching of P_0. Then $M_0 \cup M_1 \cup M_2 \cup M_{3,4} \cup \cdots \cup M_{k-2,k-1} \cup \{(y_0, y_1), (z_1, z_2)\}$ is a perfect matching in $Q_2^k - F$. Next, consider that $e \neq (y_0, y_{k-1})$. Suppose that $k = 3$. If $y_0 = x_0$, then the cycle $(u_1, u_2, v_2, v_1, u_1)$ has a perfect matching M^* containing no fault edges. Now, $M^* \cup \{(y_0, y_2)\}$ is a perfect matching in $Q_2^3 - F$. If $y_0 \neq x_0$, say $x_0 = u_0$, then the cycle $(y_0, y_1, v_1, v_2, u_2, y_2, y_0)$ has a perfect matching containing no fault edges, which is a perfect matching in $Q_2^3 - F$.

Assume that $P_1 = y_0$, $e \neq (y_0, y_{k-1})$ and $k \geq 5$ or $|P_1| \geq 3$. When $|P_1| \geq 3$, we have $k \geq 5$, and there exists a terminal vertex y_0 of P_1 such that $e \neq (y_0, y_{k-1})$. Let M_0, M_1 and M_{k-1} be the perfect matchings of $C_0 - \{u_0, v_0, y_0\}$, $C_1 - x_1$ and $C_{k-1} - y_{k-1}$, respectively. Let $M = M_0 \cup M_1 \cup M_{k-1} \cup M_{2,3} \cup \cdots \cup M_{k-3,k-2} \cup \{(y_0, y_{k-1})\}$. If $e \notin M$, then M is a perfect matching in $Q_2^k - F$. If $e \in M_{j,j+1} \subseteq M$, say $e = (a_j, a_{j+1})$, then $M \cup \{(a_j, b_j), (a_{j+1}, b_{j+1})\} \backslash \{(a_j, a_{j+1}), (b_j, b_{j+1})\}$ is a perfect matching in $Q_2^k - F$, where $b_j \in N_{C_j}(a_j)$. If $e \in M_1$ or $e \in M_{k-1}$, say $e = (a_1, b_1) \in M_1$, then $M \cup \{(a_1, a_2), (b_1, b_2), (a_3, b_3)\} \backslash \{(a_1, b_1), (a_2, a_3), (b_2, b_3)\}$ is a perfect matching in $Q_2^k - F$. Suppose that $e \in M_0$. Let V_{k-2} and V_{k-3} be the corresponding vertex sets to $V(P_1)$ in C_{k-2} and C_{k-3}, respectively. Let M_{k-2} and M_{k-3} be the perfect matchings of $C_{k-2} - (V_{k-2} \cup \{u_{k-2}, v_{k-2}\})$ and $C_{k-3} - V_{k-3}$, respectively. Then $(M_{k-1,0} \backslash \{(u_0, u_{k-1}), (v_0, v_{k-1})\}) \cup M_1 \cup M_{k-2} \cup M_{k-3} \cup M_{2,3} \cup \cdots \cup M_{k-5,k-4} \cup \{(u_{k-1}, u_{k-2}), (v_{k-1}, v_{k-2})\} \cup (\cup_{t_{k-2} \in V_{k-2}} \{(t_{k-2}, t_{k-3})\})$ is a perfect matching in $Q_2^k - F$.

Case 2.2.2. $i \notin \{1, k-1\}$. In this case, $k \geq 5$. By symmetry, say that i is even.

Assume that $|P_1| = 1$ (say $P_1 = y_0$) and $e = (y_0, y_1)$. Now, $C_0 - \{u_0, v_0, y_0\}$ can be partitioned into the set M_0 of paths of length 1. Let M_{k-1} be the perfect matching of $C_{k-1} - y_{k-1}$. Let $N_{C_i}(x_i) = \{a_i, b_i\}$. Let M_j be the matching saturating $C_j - \{a_j, b_j, x_j\}$ for each $j \in \{i-1, i, i+1\}$. Then $M_0 \cup M_{k-1} \cup M_{i-1} \cup M_i \cup M_{i+1} \cup M_{1,2} \cup \cdots \cup M_{i-3,i-2} \cup M_{i+2,i+3} \cup \cdots \cup M_{k-3,k-2} \cup \{(y_0, y_{k-1}), (a_{i-1}, a_i), (a_{i+1}, x_{i+1}), (b_i, b_{i+1}), (x_{i-1}, b_{i-1})\}$ is a perfect matching in $Q_2^k - F$.

Assume that there exists a terminal vertex y_0 of P_1 such that $e \neq (y_0, y_1)$. Let M_0, M_1 and M_i be the perfect matchings of $C_0 - \{u_0, v_0, y_0\}$, $C_1 - y_1$ and $C_i - x_i$, respectively. Let $M = M_0 \cup M_1 \cup M_i \cup M_{2,3} \cup \cdots \cup M_{i-2,i-1} \cup M_{i+1,i+2} \cup \cdots \cup M_{k-2,k-1} \cup \{(y_0, y_1)\}$. If $e \notin M$, then M is a perfect matching in $Q_2^k - F$. If $e \in M_{j,j+1} \subseteq M$, say $e = (a_j, a_{j+1})$, then $M \cup \{(a_j, b_j), (a_{j+1}, b_{j+1})\} \backslash \{(a_j, a_{j+1}), (b_j, b_{j+1})\}$ is a perfect matching in $Q_2^k - F$, where $b_j \in N_{C_j}(a_j)$. If $e \in M_0$, say $e = (a_0, b_0)$, then $M \cup \{(a_0, a_{k-1}), (b_0, b_{k-1}), (a_{k-2}, b_{k-2})\} \backslash \{(a_0, b_0), (a_{k-1}, a_{k-2}), (b_{k-1}, b_{k-2})\}$ is a perfect matching in $Q_2^k - F$. If $e \in M_i$, say $e = (a_i, b_i)$, then $M \cup \{(a_i, a_{i+1}), (b_i, b_{i+1}), (a_{i+2}, b_{i+2})\} \backslash \{(a_i, b_i), (a_{i+1}, a_{i+2}), (b_{i+1}, b_{i+2})\}$ is a perfect matching in $Q_2^k - F$. Next, suppose that $e \in M_1$, say $e = (a_1, b_1)$. If $i \neq 2$, then $M \cup \{(a_1, a_2), (b_1, b_2), (a_3, b_3)\} \backslash \{(a_1, b_1), (a_2, a_3), (b_2, b_3)\}$ is a perfect matching in $Q_2^k - F$. If $i = 2$ and $(a_2, b_2) \in M_2$, then $M \cup \{(a_1, a_2), (b_1, b_2)\} \backslash \{(a_1, b_1), (a_2, b_2)\}$ is a perfect matching in $Q_2^k - F$. If $i = 2$ and $x_2 \in \{a_2, b_2\}$, then, without loss of generality, assume that $x_2 = a_2$. Let $d_2 \in N_{C_2}(x_2)$ such that d_2 and b_2 are distinct. Let M_{k-1} be the perfect matching of $C_{k-1} - y_{k-1}$. Let M_j' be the perfect matching of $C_j - \{d_j, a_j, b_j\}$ for each $j \in \{1, 2, 3\}$. Then $M_0 \cup M_1' \cup M_2' \cup M_3' \cup M_{k-1} \cup M_{4,5} \cup \cdots \cup M_{k-3,k-2} \cup \{(y_0, y_{k-1}), (d_1, a_1), (b_1, b_2), (d_2, d_3), (a_3, b_3)\}$ is a perfect matching in $Q_2^k - F$. If $i = 2$ and $\{(a_2, c_2), (b_2, d_2)\} \subseteq M_2$, then $M \cup \{(a_1, a_2), (b_1, b_2), (c_2, c_3), (d_2, d_3), (a_3, b_3), (a_4, c_4), (b_4, d_4)\} \backslash \{(a_1, b_1), (a_2, c_2), (b_2, d_2), (c_3, c_4), (a_3, a_4), (b_3, b_4), (d_3, d_4)\}$ is a perfect matching in $Q_2^k - F$.

Case 2.3. $|F_v \cap V(C_0)| = 1$.

Let $F_v \cap V(C_0) = \{z_0\}$. Let $x_i \in V(C_i)$ and $y_j \in V(C_j)$ be the other two fault vertices, where $0 < i < j \leq k-1$. If $x_0 = y_0 = z_0$, then, similar to the proof of Case 2.1, we can obtain a perfect matching in $Q_2^k - F$. If two of x_0, y_0, and z_0 are the same, then, similar to the proof of Case 2.2, F is a trivial strong matching preclusion set or we can obtain a perfect matching in $Q_2^k - F$. Next, assume that x_0, y_0, and z_0 are three distinct vertices in C_0. Without loss of generality, assume that e is not a 1-dimensional edge.

Suppose that $C_0 - \{x_0, y_0, z_0\}$ can be partitioned into the set M_0 of paths of length 1. Let M_t be the corresponding matching to M_0 for $t = 1, 2, \ldots, k-1$. Let M_w be the matching saturating $C(w) - w$ for each $w \in \{x_i, y_j, z_0\}$. Let $M = (\cup_{t=0}^{k-1} M_t) \cup (\cup_{w \in \{x_i, y_j, z_0\}} M_w)$. Clearly, there is no fault edge in $\cup_{w \in \{x_i, y_j, z_0\}} M_w$. If $e \notin M$, then M is a perfect matching in $Q_2^k - F$. If $e \in M_t \subseteq M$, say $e = (a_t, b_t)$, then $M \cup \{(a_t, a_{t+1}), (b_t, b_{t+1})\} \backslash \{(a_t, b_t), (a_{t+1}, b_{t+1})\}$ is a perfect matching in $Q_2^k - F$.

Suppose that $C_0 - \{x_0, y_0, z_0\}$ can be partitioned into the set M_0 of paths of length 1 plus two single vertices u_0 and v_0, both of which are adjacent to one of x_0, y_0 and z_0, say x_0. Let M_t be the corresponding matching to M_0 for $t = 1, 2, \ldots, k-1$. Let M_w be the matching saturating $C(w) - w$ for each $w \in \{y_j, z_0\}$. Let M_{s_i} be the matching saturating $C(s_i) - \{s_{i-1}, s_i, s_{i+1}\}$ for each $s_i \in \{u_i, v_i, x_i\}$. Note that $C = (u_{i-1}, u_i, u_{i+1}, x_{i+1}, v_{i+1}, v_i, v_{i-1}, x_{i-1}, u_{i-1})$ is a cycle which contains at most one fault edge. So there exists a perfect matching M^* containing no fault edges of C. Let $M' = (\cup_{t=0}^{k-1} M_t) \cup M_{y_j} \cup M_{z_0} \cup (\cup_{s_i \in \{u_i, v_i, x_i\}} M_{t_0}) \cup M^*$. Clearly, there is no fault edge in $M_{y_j} \cup M_{z_0} \cup (\cup_{s_i \in \{u_i, v_i, x_i\}} M_{t_0}) \cup M^*$. If $e \notin M'$, then M' is a perfect matching in $Q_2^k - F$. If $e \in M_t \subseteq M'$, say $e = (a_t, b_t)$, then $M' \cup \{(a_t, a_{t+1}), (b_t, b_{t+1})\} \backslash \{(a_t, b_t), (a_{t+1}, b_{t+1})\}$ is a perfect matching in $Q_2^k - F$.

Case 3. $|F_v| = 2$ and $|F_e| = 2$.

By Lemma 3.2, $Q_2^k - F_v$ has a Hamiltonian cycle C. Note that $|C| = k^2 - 2$ is odd. $C - F_e$ can be partitioned into the set M of paths of length 1 plus one single vertex. Now, M is an almost perfect matching in $Q_2^k - F$, which means that $Q_2^k - F$ is matchable.

Case 4. $|F_v| = 1$ and $|F_e| = 3$.

Let $F_v \cap V(C_0) = \{x_0\}$. Without loss of generality, assume that there is at most one fault 1-dimensional edge. We consider four subcases.

Case 4.1. $|F_e \cap E(C_0)| = 3$.

Now, there is no fault 1-dimensional edge. Let M_2 be the perfect matching in $C_2 - x_2$. Then $(M_{0,1} \backslash \{(x_0, x_1)\}) \cup M_2 \cup M_{3,4} \cup \cdots \cup M_{k-2,k-1} \cup \{(x_1, x_2)\}$ is a perfect matching in $Q_2^k - F$.

Case 4.2. $|F_e \cap E(C_0)| = 2$.

Recall that no fault edge is incident to x_0. We have $k \geq 5$. Note that there is exactly one fault edge e which is not in C_0. Let M_2 and M_{k-2} be the perfect matchings of $C_2 - x_2$ and $C_{k-2} - x_{k-2}$, respectively. Whether $(M_{0,1} \backslash \{(x_0, x_1)\}) \cup M_2 \cup \{(x_1, x_2)\}$ or $(M_{k-1,0} \backslash \{(x_0, x_{k-1})\}) \cup M_{k-2} \cup \{(x_{k-1}, x_{k-2})\}$ contains no fault edges, say that $(M_{0,1} \backslash \{(x_0, x_1)\}) \cup M_2 \cup \{(x_1, x_2)\}$ contains no fault edges. Let $M = (M_{0,1} \backslash \{(x_0, x_1)\}) \cup M_2 \cup M_{3,4} \cup \cdots \cup M_{k-2,k-1} \cup \{(x_1, x_2)\}$. If $e \notin M$, then M is a perfect matching in $Q_2^k - F$. If $e \in M_{j,j+1} \subseteq M$, say $e = (a_j, a_{j+1})$, then $M \cup \{(a_j, b_j), (a_{j+1}, b_{j+1})\} \backslash \{(a_j, a_{j+1}), (b_j, b_{j+1})\}$ is a perfect matching in $Q_2^k - F$, where $b_j \in N_{C_j}(a_j)$.

Case 4.3. $|F_e \cap E(C_0)| = 1$.

Let M_2 be the perfect matching in $C_2 - x_2$. Let $M = (M_{0,1} \backslash \{(x_0, x_1)\}) \cup M_2 \cup M_{3,4} \cup \cdots \cup M_{k-2,k-1} \cup \{(x_1, x_2)\}$. Assume that there is no fault 1-dimensional edge. Without loss of generality, assume that $|F_e \cap E(C_1)| \geq |F_e \cap E(C_{k-1})|$. If $|F_e \cap E(C_1)| = 2$, then M is a perfect matching in $Q_2^k - F$. Suppose that $|F_e \cap E(C_1)| = 1$. Let e be the other fault edge. If $e \notin M_2$, then M is a perfect matching in $Q_2^k - F$. If $e \in M_2$, say $e = (a_2, b_2)$, then $M \cup \{(a_2, a_3), (b_2, b_3), (a_4, b_4)\} \backslash \{(a_2, b_2), (a_3, a_4), (b_3, b_4)\}$ is a perfect matching in $Q_2^k - F$ when $k \geq 5$, $\{(a_0, a_2), (b_0, b_2), (x_1, x_2), (a_1, b_1)\}$ is a perfect matching in $Q_2^k - F$ when $k = 3$ and (a_1, b_1) is not the fault edge in C_1, and $\{(x_1, a_1), (x_2, b_2), (a_0, a_2), (b_0, b_1)\}$ is a perfect matching in $Q_2^k - F$ when $k = 3$ and (a_1, b_1) is the fault edge in C_1. Next, suppose that $|F_e \cap E(C_1)| = 0$. Now, $|F_e \cap E(C_1)| = |F_e \cap E(C_{k-1})| = 0$, which implies that $k \geq 5$. If $e \notin M_2$, then M is a perfect matching in $Q_2^k - F$. If there are at most two fault edges (a_2, b_2) and (c_2, d_2) in M_2, then $M \cup \{(a_2, a_3), (b_2, b_3), (c_2, c_3), (d_2, d_3), (a_4, b_4), (c_4, d_4)\} \backslash \{(a_2, b_2), (c_2, d_2), (a_3, a_4), (b_3, b_4), (c_3, c_4), (d_3, d_4)\}$ is a perfect matching in $Q_2^k - F$.

Assume that there is one fault 1-dimensional edge e_1. Without loss of generality, assume that $|F_e \cap M_{0,1}| = 0$. Let e_2 be the other fault edge. If $e_1, e_2 \notin M$, then M is a perfect matching in $Q_2^k - F$. Suppose that $e_1 \notin M$ and $e_2 \in M$, say $e_2 = (a_2, b_2)$. Then $\{(a_0, a_2), (x_1, x_2), (b_0, b_2), (a_1, b_1)\}$ is a perfect matching in $Q_2^k - F$ when $k = 3$ and $M \cup \{(a_2, a_3), (b_2, b_3), (a_4, b_4)\} \backslash \{(a_2, b_2), (a_3, a_4), (b_3, b_4)\}$ is a perfect matching in $Q_2^k - F$ when $k \geq 5$. Next, consider that $e_1 \in M$. Suppose that $e_1 = (x_1, x_2)$. If $k = 3$, without loss of generality, assume that $e_2 \notin E(C_1)$; then $\{(x_1, a_1), (x_2, b_2), (a_0, a_2), (b_0, b_1)\}$ or $\{(x_1, b_1), (x_2, a_2), (a_0, a_1), (b_0, b_2)\}$ is a perfect matching in $Q_2^k - F$, where $\{a_0, b_0\} = V(C_0) \backslash \{x_0\}$. If $k \geq 5$, then there exists $a_2 \in N_{C_2}(x_2)$ such that $(a_2, b_2) \in M_2$ and (a_0, b_0) is not the fault edge. Let $M' = M \cup \{(x_1, a_1), (x_2, a_2), (b_1, b_2), (a_0, b_0)\} \backslash \{(a_0, a_1), (b_0, b_1), (x_1, x_2), (a_2, b_2)\}$. Consider that $e_2 \notin M'$; then M' is a perfect matching in $Q_2^k - F$. Consider that $e_2 \neq (x_1, a_1)$ and $e_2 \in M'$, which implies that $e_2 \in M_2$, say $e_2 = (c_2, d_2)$; then $M' \cup \{(c_2, c_3), (d_2, d_3), (c_4, d_4)\} \backslash \{(c_2, d_2), (c_3, c_4), (d_3, d_4)\}$ is a perfect matching in $Q_2^k - F$. Consider that $e_2 = (x_1, a_1)$, and denote the perfect matchings of $C_1 - a_1$, $C_2 - a_2$ and $C_{k-2} - x_{k-2}$ by M_1, M_2' and M_{k-2}, respectively; then $(M_{k-1,0} \backslash \{(x_0, x_{k-1})\}) \cup M_1 \cup M_2' \cup M_{k-2} \cup M_{3,4} \cup \cdots \cup M_{k-4,k-3} \cup \{(x_{k-1}, x_{k-2}), (a_1, a_2)\}$ is a perfect matching in $Q_2^k - F$.

Next, suppose that $e_1 \neq (x_1, x_2)$. Now, $k \geq 5$. Without loss of generality, assume that $e_1 = (a_j, a_{j+1}) \in M$. Then there exists $b_j \in N_{C_j}(a_j)$ such that (a_j, b_j) and (a_{j+1}, b_{j+1}) are not fault edges. Let $M' = M \cup \{(a_j, b_j), (a_{j+1}, b_{j+1})\} \backslash \{(a_j, a_{j+1}), (b_j, b_{j+1})\}$. If $e_2 \notin M_2$, then M' is a perfect matching in $Q_2^k - F$. Consider that $e_2 = (u_2, v_2) \in M_2$. When $\{a_j, b_j\} \cap \{u_3, v_3\} = \emptyset$, we have that $M' \cup \{(u_2, u_3), (v_2, v_3), (u_4, v_4)\} \backslash \{(u_2, v_2), (u_3, u_4), (v_3, v_4)\}$ is a perfect matching in $Q_2^k - F$. When $b_j \in \{u_3, v_3\}$ and $a_j \notin \{u_3, v_3\}$, we have that $M \cup \{(a_j, c_j), (a_{j+1}, c_{j+1}), (u_2, u_3), (v_2, v_3), (u_4, v_4)\} \backslash \{(a_j, a_{j+1}), (c_j, c_{j+1}), (u_2, v_2), (u_3, u_4), (v_3, v_4)\}$ is a perfect matching in $Q_2^k - F$, where $c_j \in N_{C_j}(a_j)$ and $c_j \neq b_j$. When $a_j \in \{u_3, v_3\}$, we have that $M \cup \{(u_2, u_3), (v_2, v_3), (u_4, v_4)\} \backslash \{(u_2, v_2), (u_3, u_4), (v_3, v_4)\}$ is a perfect matching in $Q_2^k - F$.

Case 4.4. $|F_e \cap E(C_0)| = 0$.

Note that $|F_e| = 3$ and that there is at most one fault 1-dimensional edge. We have $2 \leq \sum_{i=1}^{k-1} |F_e \cap E(C_i)| \leq 3$. Let M_0 be the perfect matching in $C_0 - x_0$. Let $M = M_0 \cup M_{1,2} \cup \cdots \cup M_{k-2,k-1}$. If $\sum_{i=1}^{k-1} |F_e \cap E(C_i)| = 3$, then M is a perfect matching in $Q_2^k - F$.

Next, assume that $\sum_{i=1}^{k-1} |F_e \cap E(C_i)| = 2$. Suppose that $k = 3$. Let $\{a_0, b_0\} = V(C_0)\backslash\{x_0\}$. Without loss of generality, assume that $|F_e \cap E(C(b_0))| = 0$ and $|F_e \cap E(C_1)| \geq |F_e \cap E(C_2)|$. If $|F_e \cap E(C_1)| = 2$, then it is easy to verify that either $Q_2^k - F$ has a perfect matching or F is a trivial strong matching preclusion set. If $|F_e \cap E(C_1)| = |F_e \cap E(C_2)| = 1$, then M, $\{(x_1, a_1), (x_2, b_2), (a_0, a_2), (b_0, b_1)\}$, $\{(x_1, b_1), (x_2, a_2), (a_0, a_1), (b_0, b_2)\}$, or $\{(x_1, x_2), (a_1, b_1), (a_0, a_2), (b_0, b_2)\}$ is a perfect matching in $Q_2^k - F$. Suppose that $k \geq 5$. Let $e = (a_j, a_{j+1})$ be the fault 1-dimensional edge. If $e \notin M$, then M is a perfect matching in $Q_2^k - F$. Consider that $e \in M$ and there exists $b_j \in N_{C_j}(a_j)$ such that (a_j, b_j) and (a_{j+1}, b_{j+1}) are not fault edges, then $M \cup \{(a_j, b_j), (a_{j+1}, b_{j+1})\}\backslash\{(a_j, a_{j+1}), (b_j, b_{j+1})\}$ is a perfect matching in $Q_2^k - F$. Consider that $e \in M$ and that there exists $b_j \in N_{C_j}(a_j)$ such that (a_j, b_j) is not the fault edge and (a_{j+1}, b_{j+1}) is a fault edge; then $M \cup \{(a_j, b_j), (a_{j+1}, a_{j+2}), (b_{j+1}, b_{j+2}), (a_{j+3}, b_{j+3})\}\backslash\{(a_j, a_{j+1}), (b_j, b_{j+1}), (a_{j+2}, a_{j+3}), (b_{j+2}, b_{j+3})\}$ is a perfect matching in $Q_2^k - F$. Consider that $e \in M$ and (a_j, b_j) and (a_j, d_j) are fault edges, where $\{b_j, d_j\} = N_{C_j}(a_j)$. If $j = 1$ and $a_0 = x_0$, then F is a trivial strong matching preclusion set. If $j = 1$ and $a_0 \neq x_0$, say $(a_0, u_0) \in M_0$, then $M \cup \{(a_0, a_1), (u_0, u_1), (a_2, u_2)\}\backslash\{(a_0, u_0), (a_1, a_2), (u_1, u_2)\}$ is a perfect matching in $Q_2^k - F$. If $j \neq 1$, then $M \cup \{(a_{j-2}, b_{j-2}), (a_{j-1}, a_j), (b_{j-1}, b_j), (a_{j+1}, b_{j+1})\}\backslash\{(a_{j-2}, a_{j-1}), (b_{j-2}, b_{j-1}), (a_j, a_{j+1}), (b_j, b_{j+1})\}$ is a perfect matching in $Q_2^k - F$.

Case 5. $|F_v| = 0$, which means that $|F_e| = 4$.

Let $F' \subseteq F$ with $|F'| = 2$. By Lemma 3.2, $Q_2^k - F'$ has a Hamiltonian cycle C. Note that $|C| = k^2$ is odd. $C - (F\backslash F')$ is divided into an even path and an odd path. So $C - (F\backslash F')$ can be partitioned into the set M of paths of length 1 plus one single vertex. Now, M is an almost perfect matching in $Q_2^k - F$, which means that $Q_2^k - F$ is matchable. □

Lemma 3.3. *Suppose that a graph G has an almost perfect matching M that misses a vertex v which is not isolated. Then there are at least $d_G(v)$ distinct almost perfect matchings other than M with distinct vertices missed other than v.*

Proof. Since v is not isolated, there is another vertex $u \in N_G(v)$. Let $(u, w) \in M$. Then $M \cup \{(u, v)\}\backslash\{(u, w)\}$ is an almost perfect matching in G with w missed. According to the above discussion, the lemma is clearly true. □

For some $d \in \{1, 2, \ldots, n\}$, a Q_n^k can be divided into $Q[0], Q[1], \ldots, Q[k-1]$ along dimension d. For $0 \leq i, j \leq k-1$, we use $[i, j]$ to denote a set of integers: $[i, j] = \{l : i \leq l \leq j\}$ if $i \leq j$, and $[i, j] = \{l : i \leq l \leq k-1 \text{ or } 0 \leq l \leq j\}$ if $i > j$. $Q_n^k[i, j]$ (abbreviated as $Q[i, j]$ if there is no ambiguity) denotes the subgraph of Q_n^k which is induced by $\{u : u \in V(Q[l]), l \in [i, j]\}$.

Lemma 3.4 ([20]). *Let $i, j \in [0, k-1]$, and let $F \subseteq V(Q[i, j]) \cup E(Q[i, j])$ be a faulty set with $|F| \leq 2n-2$. For any $l \in [i, j]$, let $F_l = F \cap (V(Q[l]) \cup E(Q[l]))$. If $Q[l] - F_l$ is Hamiltonian connected for every $l \in [i, j]$, then there exists a Hamiltonian path connecting two arbitrary vertices $u_i \in V(Q[i] - F_i)$ and $v_j \in V(Q[j] - F_j)$ in $Q[i, j] - F$ for every $n \geq 3$ and odd $k \geq 3$.*

Before we prove the strong matching preclusion result for Q_n^k with its fault set F, we define an approach to find a perfect matching or an almost perfect matching in $Q_n^k - F$ as follows: we find a fault-free matching M saturating a subset $V \subseteq V(Q_n^k)$. If there exists a fault-free Hamiltonian path P in $Q_n^k - V$, then we can extend the matching M to a perfect matching or an almost perfect matching in $Q_n^k - F$ by adding a perfect matching or an almost perfect matching in P. This method will be called completing M with P.

Theorem 3.5. *Let $k \geq 3$ be an odd integer, and let $n \geq 2$ be an integer. Then Q_n^k is super strong matched.*

Proof. The statement is true if $n = 2$ by Theorem 3.4. Next, assume that $n \geq 3$. We proceed by induction on n. Suppose that Q_{n-1}^k is super strong matched. By Theorem 3.3, $smp(Q_n^k) = 2n$. Let $F = F_v \cup F_e$ be a fault set in Q_n^k such that $|F| = 2n$, where F_v and F_e are the fault vertex set and the fault edge set, respectively. To prove our main result, it is enough to show that either $Q_n^k - F$ is matchable or F is a trivial strong matching preclusion set, where no fault edge in F is incident to any fault vertex in F.

Claim 1. If $|F_v| = 0$ or $|F_e| = 0$, then $Q_n^k - F$ is matchable.

Assume that $|F_v| = 0$ or $|F_e| = 0$. Let $F' \subseteq F$ with $|F'| = 2n-2$. By Lemma 3.2, $Q_n^k - F'$ has a Hamiltonian cycle C. Note that $C - (F\backslash F')$ can be divided into an even path and an odd path. We have that $C - (F\backslash F')$ can be partitioned into the set M of paths of length 1 plus one single vertex. Now, M is an almost perfect matching in $Q_n^k - F$, which means that $Q_n^k - F$ is matchable. The proof of Claim 1 is complete.

By Claim 1, if $|F_v| = 0$ or $|F_e| = 0$, then the conclusion is true. Next, we only consider the case that $1 \leq |F_v| \leq 2n-1$ and $1 \leq |F_e| \leq 2n-1$. There exists $d \in \{1, 2, \ldots, n\}$ such that there is at least one fault d-dimensional edge. Q_n^k can be divided into $Q[0], Q[1], \ldots, Q[k-1]$ along dimension d. Let $F_i = F \cap (V(Q[i]) \cup E(Q[i]))$ for $i = 0, 1, \ldots, k-1$. Without loss of generality, assume that $|F_0| \geq |F_i|$ for $i = 1, 2, \ldots, k-1$. We consider five cases, depending on the value of $|F_0|$.

Case 1. $|F_0| = 2n-1$.

Note that $1 \leq |F_v| \leq 2n-1$. Let $v_0 \in F_0$ be a fault vertex. Let $F_0' = F_0\backslash\{v_0\}$. Note that $Q[0]$ is isomorphic to Q_{n-1}^k and $|F_0'| = 2(n-1)$. By the induction hypothesis, $Q[0] - F_0'$ is matchable or F_0' is a trivial strong matching preclusion set in $Q[0]$. Recall that there is exactly one fault d-dimensional edge. By Lemma 3.2, $Q[l] - F_l$ is Hamiltonian connected for every $l \in [1, k-1]$. We consider two subcases.

Case 1.1. $|V(Q[0]-F_0')|$ is odd.

In this case, F_0' cannot be a trivial strong matching preclusion set in $Q[0]$. So $Q[0]-F_0'$ has an almost perfect matching M that misses w_0. Let $(u_0, v_0) \in M$. Recall that there is exactly one fault d-dimensional edge. We have that $\{(u_0, u_1), (w_0, w_{k-1})\} \cap F = \emptyset$ or $\{(u_0, u_{k-1}), (w_0, w_1)\} \cap F = \emptyset$. Without loss of generality, assume that $\{(u_0, u_1), (w_0, w_{k-1})\} \cap F = \emptyset$. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting u_1 and w_{k-1} in $Q[1, k-1]-F$. Let $P = P_0 \cup \{(u_0, u_1), (w_0, w_{k-1})\}$. Then completing $M \backslash \{(u_0, v_0)\}$ with P gives a perfect matching of $Q_n^k - F$.

Case 1.2. $|V(Q[0]-F_0')|$ is even.

Suppose that $Q[0]-F_0'$ has a perfect matching M. Let $(u_0, v_0) \in M$. Recall that there is exactly one fault d-dimensional edge. We have that either (u_0, u_1) or (u_0, u_{k-1}) is not a fault edge. Without loss of generality, assume that (u_0, u_1) is not a fault edge. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting u_1 and u_{k-1} in $Q[1, k-1]-F$. Let $P = P_0 \cup \{(u_0, u_1)\}$. Then completing $M \backslash \{(u_0, v_0)\}$ with P gives an almost perfect matching of $Q_n^k - F$.

Suppose that F_0' is a trivial strong matching preclusion set in $Q[0]$. Then there exists $v_0' \in F_0'$ such that v_0' is another fault vertex. Let $F_0'' = F_0 \backslash \{v_0'\}$. Now, F_0'' cannot be a trivial strong matching preclusion set in $Q[0]$. So $Q[0]-F_0''$ has a perfect matching. Similar to the discussion in the above paragraph, we can obtain an almost perfect matching of $Q_n^k - F$.

Case 2. $|F_0| = 2n-2$.

In this case, $|F_l| \le 1$ for every $l \in [1, k-1]$. By Lemma 3.2, $Q[l]-F_l$ is Hamiltonian connected for every $l \in [1, k-1]$. By the induction hypothesis, $Q[0]-F_0$ is matchable or F_0 is a trivial strong matching preclusion set in $Q[0]$. We consider two subcases.

Case 2.1. $Q[0]-F_0$ is matchable.

Assume that $Q[0]-F_0$ has a perfect matching M. Note that $|F \backslash F_0| = 2 \le 2n-2$. By Lemma 3.4, there exists a Hamiltonian path P in $Q[1, k-1]-F$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$.

Assume that $Q[0]-F_0$ has an almost perfect matching M that misses x_0. Suppose that at least one of (x_0, x_{k-1}) and (x_0, x_1) is not faulty. Without loss of generality, assume that (x_0, x_1) is not faulty, and so x_1 is not a fault vertex in $Q[1]-F_1$. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting x_1 and u_{k-1} in $Q[1, k-1]-F$, where $u_{k-1} \in V(Q[k-1]-F_{k-1})$. Let $P = P_0 \cup \{(x_0, x_1)\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$. Suppose that both (x_0, x_{k-1}) and (x_0, x_1) are faulty. We first consider the case that x_0 is not isolated in $Q[0]-F_0$. By Lemma 3.3, there exists an almost perfect matching in $Q[0]-F_0$ that misses a vertex other than x_0. Similar to the above discussion, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$. Next, we consider the case that x_0 is isolated in $Q[0]-F_0$. Since $Q[0]-F_0$ has an almost perfect matching, we have that $|V(Q[0]-F_0)|$ is odd. If either $F \backslash F_0 = \{x_1, (x_0, x_{k-1})\}$ or $F \backslash F_0 = \{(x_0, x_1), x_{k-1}\}$ holds, then F is a trivial strong matching preclusion set. Otherwise, $F \backslash F_0 = \{(x_0, x_1), (x_0, x_{k-1})\}$. By Lemma 3.4, there exists a Hamiltonian path P in $Q[1, k-1]-F$. Now, P is an odd path, and so P has a perfect matching M_p. Then $M \cup M_p$ gives an almost perfect matching of $Q_n^k - F$.

Case 2.2. F_0 is a trivial strong matching preclusion set in $Q[0]$.

Let x_0 be isolated in $Q[0]-F_0$. Now, $|V(Q[0]-F_0)|$ is even, and there exists a fault vertex $v_0 \in F_0$. Let $F_0' = F_0 \cup \{x_0\} \backslash \{v_0\}$. Then F_0' is not a trivial strong matching preclusion set in $Q[0]$. By the induction hypothesis, $Q[0]-F_0'$ has a perfect matching M. Let $(v_0, u_0) \in M$.

Suppose that both (x_0, x_{k-1}) and (x_0, x_1) are faulty. If $F \backslash F_0 = \{(x_0, x_1), (x_0, x_{k-1})\}$, then F is a trivial strong matching preclusion set. Otherwise, either $F \backslash F_0 = \{x_1, (x_0, x_{k-1})\}$ or $F \backslash F_0 = \{(x_0, x_1), x_{k-1}\}$ holds. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting u_1 and w_{k-1} in $Q[1, k-1]-F$, where $w_{k-1} \in V(Q[k-1]-F_{k-1})$. Let $P = P_0 \cup \{(u_0, u_1)\}$. Then P is an odd path, and so P has a perfect matching M_p. Then $M \cup M_p \backslash \{(v_0, u_0)\}$ gives an almost perfect matching of $Q_n^k - F$.

Suppose that at least one of (x_0, x_{k-1}) and (x_0, x_1) is not faulty. Without loss of generality, assume that (x_0, x_1) is not faulty, and so x_1 is not a fault vertex in $Q[1]-F_1$. We first consider the case that there exists exactly one fault vertex in $F \backslash F_0$. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting x_1 and w_{k-1} in $Q[1, k-1]-F$, where $w_{k-1} \in V(Q[k-1]-F_{k-1})$. Let $P = P_0 \cup \{(x_0, x_1)\}$. Then P is an odd path, and so P has a perfect matching M_p. Then $M \cup M_p \backslash \{(v_0, u_0)\}$ gives an almost perfect matching of $Q_n^k - F$. Next, consider the case that $F \backslash F_0$ contains no fault vertices. Let u_0' be adjacent to u_0 in $Q[0]-F_0$ such that $u_0' \neq x_0$, and let $(u_0', v_0') \in M$. Assume that (u_0, u_{k-1}) is not faulty. Let P_0 be a Hamiltonian path connecting x_1 and u_{k-1}, and let $P = P_0 \cup \{(x_0, x_1), (u_0, u_{k-1})\}$. Then completing $M \backslash \{(u_0, v_0)\}$ with P gives a perfect matching of $Q_n^k - F$. Assume that (u_0, u_{k-1}) is faulty and that (v_0', v_{k-1}') is not faulty. Let P_0 be a Hamiltonian path connecting x_1 and v_{k-1}', and let $P = P_0 \cup \{(x_0, x_1), (v_0', v_{k-1}')\}$. Then completing $M \cup \{(u_0, u_0')\} \backslash \{(u_0, v_0), (u_0', v_0')\}$ with P gives a perfect matching of $Q_n^k - F$. Assume that both (u_0, u_{k-1}) and (v_0', v_{k-1}') are faulty. Let P_0 be a Hamiltonian path connecting u_1 and x_{k-1}, and let $P = P_0 \cup \{(u_0, u_1), (x_0, x_{k-1})\}$. Then completing $M \backslash \{(u_0, v_0)\}$ with P gives a perfect matching of $Q_n^k - F$.

Case 3. $|F_0| = 2n-3$.

By the induction hypothesis, $Q[0]-F_0$ is matchable. We consider two subcases.

Case 3.1. $Q[0]-F_0$ has a perfect matching M.

Assume that $|F_l| \le 1$ for every $l \in [1, k-1]$. By Lemma 3.2, $Q[l]-F_l$ is Hamiltonian connected for every $l \in [1, k-1]$. Note that $|F \backslash F_0| = 3 \le 2n-2$. By Lemma 3.4, there exists a Hamiltonian path P in $Q[1, k-1]-F$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$.

Assume that there exists some $j \in [1, k-1]$ such that $|F_j| = 2$. Note that $|F_j| = 2 \le 2(n-1)-2$. By Lemma 3.2, $Q[j]-F_j$ has a Hamiltonian cycle C and $Q[l]-F_l$ is Hamiltonian connected for every $l \in [1, k-1] \backslash \{j\}$. Suppose that $j \in \{1, k-1\}$; without loss of generality, assume that $j = 1$. Recall that there is exactly one fault d-dimensional edge. There exists $(u_1, w_1) \in E(C)$ such that (u_1, u_2) is not faulty. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting u_2 and z_{k-1} in $Q[2, k-1] - F$, where $z_{k-1} \in V(Q[k-1] - F_{k-1})$. Let $P = P_0 \cup (C - \{(u_1, w_1)\}) \cup \{(u_1, u_2)\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$. Suppose that $j \notin \{1, k-1\}$. Let $(u_j, w_j) \in E(C)$ such that both (u_j, u_{j-1}) and (w_j, w_{j+1}) are not faulty. By Lemma 3.4, there exists a Hamiltonian path P_1 connecting x_1 and u_{j-1} in $Q[1, j-1] - F$, and there exists a Hamiltonian path P_2 connecting w_{j+1} and y_{k-1} in $Q[j+1, k-1] - F$, where $x_1 \in V(Q[1] - F_1)$ and $y_{k-1} \in V(Q[k-1] - F_{k-1})$. Let $P = P_1 \cup P_2 \cup (C - \{(u_j, w_j)\}) \cup \{(u_j, u_{j-1}), (w_j, w_{j+1})\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$.

Case 3.2. $Q[0] - F_0$ has an almost perfect matching M that misses v_0.

Assume that $|F_l| \leq 1$ for every $l \in [1, k-1]$. By Lemma 3.2, $Q[l] - F_l$ is Hamiltonian connected for every $l \in [1, k-1]$. Suppose that at least one of (v_0, v_{k-1}) and (v_0, v_1) is not faulty. Without loss of generality, assume that (v_0, v_1) is not faulty, and so v_1 is not a fault vertex in $Q[1] - F_1$. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting v_1 and z_{k-1} in $Q[1, k-1] - F$, where $z_{k-1} \in V(Q[k-1] - F_{k-1})$. Let $P = P_0 \cup \{(v_0, v_1)\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$. Suppose that both (v_0, v_{k-1}) and (v_0, v_1) are faulty. Note that v_0 is not isolated in $Q[0] - F_0$. By Lemma 3.3, there exists an almost perfect matching M' in $Q[0] - F_0$ that misses a vertex x_0 other than v_0. Now, either (x_0, x_{k-1}) or (x_0, x_1) is not faulty. Similar to the above discussion, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$.

Assume that there exists some $j \in [1, k-1]$ such that $|F_j| = 2$. By Lemma 3.2, $Q[j]-F_j$ has a Hamiltonian cycle C and $Q[l]-F_l$ is Hamiltonian connected for every $l \in [1, k-1]\backslash\{j\}$. Suppose that $j \in \{1, k-1\}$; without loss of generality, assume that $j = 1$. Consider the case that (v_0, v_{k-1}) is not faulty, and so v_{k-1} is not a fault vertex in $Q[k-1]-F_{k-1}$. There exists $(u_1, w_1) \in E(C)$ such that (u_1, u_2) is not faulty. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting u_2 and v_{k-1} in $Q[2, k-1] - F$. Let $P = P_0 \cup (C - \{(u_1, w_1)\}) \cup \{(u_1, u_2), (v_0, v_{k-1})\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$. Consider the case that (v_0, v_{k-1}) is faulty. By Lemma 3.3, there exists an almost perfect matching M' in $Q[0]-F_0$ that misses a vertex x_0 other than v_0. Note that there is exactly one fault d-dimensional edge. We have that (x_0, x_{k-1}) is not faulty. Similar to the above discussion, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$.

Next, suppose that $j \notin \{1, k-1\}$. Either (v_0, v_1) or (v_0, v_{k-1}) is not faulty. Without loss of generality, assume that (v_0, v_1) is not faulty, and so v_1 is not a fault vertex in $Q[1] - F_1$. Let $(u_j, w_j) \in E(C)$ such that both (u_j, u_{j-1}) and (w_j, w_{j+1}) are not faulty. By Lemma 3.4, there exists a Hamiltonian path P_1 connecting v_1 and u_{j-1} in $Q[1, j-1]-F$, and there exists a Hamiltonian path P_2 connecting w_{j+1} and y_{k-1} in $Q[j+1, k-1]-F$, where $y_{k-1} \in V(Q[k-1]-F_{k-1})$. Let $P = P_1 \cup P_2 \cup (C - \{(u_j, w_j)\}) \cup \{(u_j, u_{j-1}), (w_j, w_{j+1}), (v_0, v_1)\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$.

Case 4. $|F_0| = 2n - 4$.

By the induction hypothesis, $Q[0] - F_0$ is matchable. Note that there is at least one fault d-dimensional edge. We have that there is at most one of $|F_1|, |F_2|, \ldots, |F_{k-1}|$ which is at least 2. We consider three subcases.

Case 4.1. There exists some $j \in [1, k-1]$ such that $|F_j| = 3$.

Recall that $|F_0| \geq |F_i|$ for every $i \in [1, k-1]$. Since $2n - 4 = |F_0| \geq |F_j| = 3$, we have that $n \geq 4$. By Lemma 3.2, $Q[l] - F_l$ is Hamiltonian connected for every $l \in [1, k-1]$. Note that $|F\backslash F_0| = 4 \leq 2n-2$, and that there is exactly one fault d-dimensional edge. Similar to the proof of the first paragraph of Case 3.1 or the first paragraph of Case 3.2, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$.

Case 4.2. There exists some $j \in [1, k-1]$ such that $|F_j| = 2$.

Assume that $Q[0] - F_0$ has a perfect matching M. Similar to the proof of the second paragraph of Case 3.1, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$.

Assume that $Q[0] - F_0$ has an almost perfect matching M that misses v_0. By Lemma 3.2, $Q[j] - F_j$ has a Hamiltonian cycle C and $Q[l] - F_l$ is Hamiltonian connected for every $l \in [1, k-1]\backslash\{j\}$. Suppose that $j \in \{1, k-1\}$; without loss of generality, assume that $j = 1$. Consider the case that (v_0, v_{k-1}) is not faulty, and so v_{k-1} is not a fault vertex in $Q[k-1] - F_{k-1}$. There exists $(u_1, w_1) \in E(C)$ such that (u_1, u_2) is not faulty. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting u_2 and v_{k-1} in $Q[2, k-1] - F$. Let $P = P_0 \cup (C - \{(u_1, w_1)\}) \cup \{(u_1, u_2), (v_0, v_{k-1})\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$. Consider the case that (v_0, v_{k-1}) is faulty. Note that there are at most two fault d-dimensional edges and that $d_{Q[0]-F_0}(v_0) \geq 2$. By Lemma 3.3, there exists an almost perfect matching M' in $Q[0] - F_0$ that misses a vertex x_0 such that $x_0 \neq v_0$ and (x_0, x_{k-1}) is not faulty. Similar to the above discussion, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$.

Next, suppose that $j \notin \{1, k-1\}$. Consider the case that either (v_0, v_1) or (v_0, v_{k-1}) is not faulty. Without loss of generality, assume that (v_0, v_1) is not faulty, and so v_1 is not a fault vertex in $Q[1] - F_1$. Let $(u_j, w_j) \in E(C)$ such that both (u_j, u_{j-1}) and (w_j, w_{j+1}) are not faulty. By Lemma 3.4, there exists a Hamiltonian path P_1 connecting v_1 and u_{j-1} in $Q[1, j-1] - F$, and there exists a Hamiltonian path P_2 connecting w_{j+1} and y_{k-1} in $Q[j+1, k-1] - F$, where $y_{k-1} \in V(Q[k-1] - F_{k-1})$. Let $P = P_1 \cup P_2 \cup (C - \{(u_j, w_j)\}) \cup \{(u_j, u_{j-1}), (w_j, w_{j+1}), (v_0, v_1)\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$. Consider the case that both (v_0, v_1) and (v_0, v_{k-1}) are faulty. By Lemma 3.3, there exists an almost perfect matching M' in $Q[0] - F_0$ that misses a vertex x_0 other than v_0. Now, (x_0, x_1) is not faulty. Similar to the above discussion, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$.

Case 4.3. $|F_l| \leq 1$ for every $l \in [1, k-1]$.

By Lemma 3.2, $Q[l] - F_l$ is Hamiltonian connected for every $l \in [1, k-1]$. Assume that $Q[0] - F_0$ has a perfect matching. Similar to the proof of the first paragraph of Case 3.1, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$. Next, assume that $Q[0] - F_0$ has an almost perfect matching M that misses v_0. Suppose that at least one of (v_0, v_{k-1}) and

(v_0, v_1) is not faulty. Without loss of generality, assume that (v_0, v_1) is not faulty, and so v_1 is not a fault vertex in $Q[1]-F_1$. By Lemma 3.4, there exists a Hamiltonian path P_0 connecting v_1 and z_{k-1} in $Q[1, k-1]-F$, where $z_{k-1} \in V(Q[k-1]-F_{k-1})$. Let $P = P_0 \cup \{(v_0, v_1)\}$. Then completing M with P gives a perfect matching or an almost perfect matching of $Q_n^k - F$. Suppose that both (v_0, v_{k-1}) and (v_0, v_1) are faulty. Note that v_0 is not isolated in $Q[0]-F_0$ and $d_{Q[0]-F_0}(v_0) \geq 2$. By Lemma 3.3, there exists an almost perfect matching M' in $Q[0] - F_0$ that misses a vertex x_0 such that $x_0 \neq v_0$ and either (x_0, x_{k-1}) or (x_0, x_1) is not faulty. Similar to the above discussion, we can obtain a perfect matching or an almost perfect matching of $Q_n^k - F$.

Case 5. $|F_0| \leq 2n - 5$.

In this case, $|F_l| \leq 2n-5 = 2(n-1)-3$ for every $l \in [0, k-1]$. Recall that $Q[l]$ is isomorphic to Q_{n-1}^k for every $l \in [0, k-1]$. By Lemma 3.2, $Q[l] - F_l$ is Hamiltonian connected for every $l \in [0, k-1]$. Now, there exist $0 \leq i \leq j \leq k-1$ such that $|F \cap (V(Q[i,j]) \cup E(Q[i,j]))| \leq 2n-2$ and $|F \cap (V(Q[j+1, i-1]) \cup E(Q[j+1, i-1]))| \leq 2n-2$. Note that $k^n > 2n$. There exists $v_i \in V(Q[i]-F_i)$ such that v_i, v_{i-1} and (v_i, v_{i-1}) are not faulty. By Lemma 3.4, there exists a Hamiltonian path P_1 connecting v_i and u_j in $Q[i,j]-F$, and there exists a Hamiltonian path P_2 connecting v_{i-1} and w_{j+1} in $Q[j+1, i-1]-F$, where $u_j \in V(Q[j]-F_j)$ and $w_{j+1} \in V(Q[j+1]-F_{j+1})$. Then $P_1 \cup P_2 \cup \{(v_i, v_{i-1})\}$ is a Hamiltonian path in $Q_n^k - F$, and so $Q_n^k - F$ is matchable. $\square$

4. Conclusion

In this paper, we have studied the strong matching preclusion for k-ary n-cubes. We have established the strong matching preclusion number and all possible minimum strong matching preclusion sets for k-ary n-cubes with $n \geq 2$ and $k \geq 3$. The results can be used in robustness analysis for the k-ary n-cube network with respect to the property of having a perfect matching or an almost perfect matching.

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